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Cover Letter

Dear Sir,

My name is hemant pandey and I am a young researcher.

I am proposing a paper on Topology of higher manifolds for possible publication. The main argument used in the paper is extension of loop shrinking property from one manifold to a higher manifold.

This is to certify that the work is original and has not been submitted any where else for publication consideration.

Details are in the pdf attachment.

Sincerely,

Hemant Pandey

2nd April 2007

Topology of higher manifolds

Abstract: Henri Poincare (1854-1912) was possibly the best person in the mathematical history of topology of manifolds that introduced a novice concept of 3-manifolds and extended it to n- manifolds. In this present paper we have attempted to derive a general method which can be applied to all manifolds in a uniform way to prove Poincare's conjecture in a general sense.

Key words: *Manifold, Poincare, 3-sphere, Topology, Loop Shrinking Property, Hyper sphere.*

Definitions:

Manifolds: ***A 2-manifold is a 3-dimesional object. Similarly 3-manifold is a 4-dimesional imaginary object etc.***

3-Sphere: ***3-sphere is the corresponding sphere of a 3-manifold. Here sphere is w.r.t. a 2-manifold.***

Topology: ***Branch of mathematics in which relative closeness is the measure of distance between two points.***

Loop shrinking property: ***This means that if a loop around a surface shrinks to point then it is topologically equivalent to a sphere in a 2-manifold.***

Hyper sphere: ***Sphere of higher dimensions.***

Hyper plane: ***Plane of higher dimensions***

Meaning of Symbols:

LSP - Loop shrinking property

n- manifold – n dimensional mathematical object.

Manuscript

Poincare’s conjecture says that assertive loop shrinking property or LSP for any 3-manifold implies it is topologically equivalent to the corresponding 3-sphere.We have attempted to find a proof that works on the method of extension from 2-manifold to 3-manifold.In general our method works from extension from (n-1) - manifold to n- manifold.

Henri Poincare was possibly the best person in the mathematical history of topology of manifolds that introduced a novice concept of 3-manifolds and extended it to n- manifolds.
His famous conjecture about 3-manifold (which has been shortly resolved by Perelman) was one of the pivotal problems of topology.
The proofs for 3- manifolds, 4- manifolds and 5th manifolds are all using different techniques.
We have tried to present a general proof that resolves the conjecture for all manifolds and hence establishes using single technique, the theorem for all manifolds.

1. *Preview*

Before we state and prove our before said technique and theorem let us have a preview of what it is really all about.
Let us start with a development of manifolds. A 1- manifold is by definition a 2 dimensional object. It is an object that has two dimensions that we usually call

length and breadth. Our conceptualization of proof i.e. general proof starts at this very basic point.

We start by asking a simple question. What this so-called 1- manifold made up of?

Mathematically it comprises of one plane. Our conceptualization says that a 1- manifold is a summation or integration of infinite 1-dimensional lines. This is a basic and valid concept so a 1- manifold is an integration of infinite 1- dimensional lines.

Let us state it as result one.

***Result 1:** A 2- dimensional plane is an integration of infinite one- dimensional lines.*

Now we move further to a higher dimension.

So if we extend our previous theorem then consequently a 2- manifold is nothing but an integration rather systematic integration of 1- manifolds or 2-D planes.

There is a profound need to appreciate this extension. Since mathematical patterns are systematic & regular we must be able to extend our previously developed concepts so as to assume evenness.

Let us delve it deeper and better.

What we have just told that a 2- manifold comprises of infinite 1- manifolds.

Earlier a 1- manifold was a union (systematic) of infinite 1- dimensional objects, i.e. Lines.

So a 1- manifold = Σ 0- manifolds.

So a 2- manifold= Σ 1- manifolds.

If we extend our above definition we could write

A n-manifold = Σ (n-1) manifolds

The above basic assumption follows from the definition of manifolds in general.

The basic axiomatic result that we would extensively use in our proof is

Result A: *n- manifold= Σ (n-1) manifolds.*

The Henri Poincare's conjecture

Poincare's conjecture says that assertive loop shrinking property implies smoothness or equivalence with corresponding hyper sphere of that dimension.

In particular if a 3- manifold assumes loop-shrinking property then it is topologically equivalent to a 3- sphere.

We would prove the above general theorem for 3- manifolds in particular and show it to hold good for n- manifolds in general.

Basic concepts: Before we start our proof we develop few basic concepts. The most basic concept is the concept of loop shrinking property or LSP in short.

It we assume the conjecture to be true for 2- manifolds (which is an established result) we can easily state and prove our new extension Theorem of 3 or higher manifolds.

Let us start with a 2- manifold. Consider a general object. It is a regular 3-dimensional distorted sphere in fig.1.

Space for fig.1

Note that above is a 3- dimensional regular 2- manifold.

As stated before it is made up of infinite 2-D planes. We may slice these parallel planes along any angle. An example is shown below in fig.2... For simplicity we have cut it parallel to the plane. However we can cut it through any angle.

Space for fig.2

Integration of Infinite such planes comprise of the object and if we go deeper we will see that each such plane contains a 2- dimensional curve, which is a part of the surface of 3-D object.

An example is given below in fig.3.

Space for fig.3

Above is cross section of a plane that is cut through a sphere. Note the part of sphere that is in the plane is merely a circle. So a sphere is a collection or union of infinitive such planes, containing circles of decreasing or increasing sizes, which represent cross section of that particular part.

On similar grounds we may consider an arbitrary object cut at any arbitrary angle to comprise of infinite planes.

Any arbitrary plane may look like fig. 4.

Space for fig. 4

Now we will develop the concept of loop shrinking property on the grounds of slice or plane model.

It we just forget the boundaries of plane (i.e. square lines) and concentrate on the periphery of the slice of one-dimensional curve we may appreciate the LSP better.

Suppose our loop shrinks perpendicular to this plane, and along the peripheries of the infinite such slices of which our slice-plane is a part. This is clearly exhibited in fig.5.

It follows instantly that if the loop shrinks to a point along the surface perpendicular to the plane then it must and should shrink to a point along this slice plane.

If one such plane obstructs implies loop cannot shrink to a point along the surface.

The reason is all but obvious. The above condition implies that there is a point on the surface of the object for which the LSP does not hold.

Above is disintegration of LSP. It just means that if a loop cannot shrink to a point along peripheries of a slice plane cross section then it cannot shrink for the object as well.

After all LSP implies that every point of the loop should shrink to a point. We may state disintegration of LSP theorem:

2. Theorem: Disintegration theorem for LSP

LSP for any manifolds means that every point of the loop traces a continuous curve path.

Proof: We may prove the above theorem through contradiction. Suppose there is a point on the loop which does not has a continuous curve implies that there is a point on the surface for which the loop breaks.

We have tried to show it by fig. 6.

Space for fig.6

Obviously it we stop at any point of the loop will not shrink or move further and will break.

We now state disintegration Theorem for manifolds.

Theorem: Disintegration Theorem for manifolds

The Theorem states that n-manifolds are a collection of infinite 'n-1'-manifolds and LSP implies that it necessary holds for 'n-1'-manifolds.

The Theorem follows at once form the arguments given before and from the definition of manifolds.

We can prove the above theorem by contradiction.

Consider a cross section of a 2-manifold.

Suppose the loop shrinks along x-axis.

It would shrink along cross-section slice of our loops that implies that it would break in the process of further shrinking if there is a stop or a point for which LSP doesn't hold.

3. Theorem: Dimension reduction theorem for manifolds

The theorem states that n-manifolds have LSP if it's corresponding 'n-1'manifolds have LSP.

The proof follows form theorem 1 and theorem 2.

We can prove it through contradiction as before.

Lemma *A direct consequence of above is that LSP for n-manifolds is characterized by LSP for its corresponding 'n-1' manifold.*

As in 2-manifold by cutting or slicing it by infinite planes one dimension is effectively reduced. Hence in an n-manifold are slicing it down by corresponding infinite hyper planes can effectively reduce dimension.

The following two results are consequence of above three theorem and lemma.

4. Particular cases and applications of previous development

Poincare's Original Conjecture – a Veracity

Henri Poincare (1854-1912) raised a query about 3-manifolds in 1900 that whether or not loop shrinking property (LSP) for a 3-manifold implies that the manifold is equivalent to a corresponding 3-Sphere

The proposition holds good for 2-D manifolds & manifolds of higher dimensions other than 3.

The following proof tries to settle the query by the method of extension from 2-manifold to 3-manifolds. The present proof is for 3-manifold but the method is equally applicable to higher dimensions.

Poincare's conjecture says that assertive loop shrinking property or LSP for any 3-manifold implies it is topologically equivalent to the corresponding 3-sphere.

The present proof works on the method of extension from 2-manifold to 3-manifold.

Poincare's Conjecture ***Assertive LSP for a 3-manifold implies its equivalence with 3-sphere***

We will start the proof from 2-manifold and extend it to 3-manifolds.

We will start our proof with a result

Result1: For a manifold of 'n'-dimensions the corresponding loop shrinks along surface equivalent to 'n-1'-dimensions.

We would take the simplest case of a 2-manifold. The LSP asserts that if the loop around the given figure shrinks to a point implies that the object is smooth or topologically equivalent to a sphere.

For simplicity we take a geometrical object.

If the given loop shrinks to point for whole object means that any plane cut through the object has LSP along its boundaries.

It implies that if an object of 'n'- dimensions has LSP the loop equivalently shrinks around corresponding infinite planes of the object in 'n-1'-dimensions

Breaking up the solid in infinite planes can always reduce one dimension and applying LSP separately to each solid.

Now if a Loop shrinks around a plane to a point implies that it is topologically Equivalent to a circle and infinite circles of that kind form a sphere.

Result 2: An object of 'n' dimensions has LSP if & only if corresponding hyper planes of 'n-1'-dimensions assume LSP.

Now we know that for all 2-manifolds LSP holds. Assuming it to hold for ‘n-1’manifolds we can easily show that it holds for ‘n+1’- manifolds as for LSP to hold for ‘n+1’- manifolds implies it holds for ‘n’-manifolds which is assumed to be true. Hence by induction the result is proved for all ‘n’.

In particular we would establish it for a 3-manifold & hence developed a method of visualization of a 3-manifold in particular & n-manifolds in general.

Result 1 : *For a 2 – manifold or a 3- dimensional object LSP implies that it shrinks to a point along the boundaries of all the planes cut through the 2-manifold.*

For clarification consider an arbitrary 2 – manifold

For simplicity we have chosen a geometrical object. The object may be thought as an integration of infinite planes that may be drawn through it in any arbitrary manner. It just means that we slice down the object in many directions and get infinite parallel planes. We can have infinite directions.

Now what we are trying to prove is that if every loop shrinks to a point along the surface of the manifold, directly implies that it shrinks to a point along the boundaries of each of the infinite planes. Because if it does not shrink to a point along any boundary of any one plane implies that there is a point on the surface of the plane where LSP does not assume. After all, the entire surface is nothing but a collection of infinite planes that can be drawn through it.

It has a nice significance Because it straight forwardly means that if LSP holds for all one dimensional surfaces it does hold for all 2 - manifolds as said before a 2- dimensional surface is nothing but a collection of infinite 1- dimensional planes.

For clarification consider a usual dumb-bell shape shown in fig 7. Obviously LSP does not hold for it at the center, P&P1

Space for fig.7

It just means that if for a moment we neglect the width of the object and just concentrate on the plane through PQP1Q1, the LSP does not holds along its boundaries, i.e. it does not holds along the contour PQP1Q1’ of the plane PP1QQ1. We are just trying to establish that all the contours of loops along a

2- D surface may be deemed as a boundary of a plane cut through it and LSP must hold for that contour.

∴ We draw infinite contours on a 2- manifolds surface. Cut infinite planes one for each contour, and see whether or not LSP holds for the contour.

It should however note that contours are path followed by each point of the loop. The basic point of above proof is dividing the corresponding hyper surface of n can always reduce that one dimension – manifold to corresponding hyper planes of n-1 dimensions and applying LSP to each of these hyper planes.

Hence if LSP assumes for all ‘n-1’ manifold $\Rightarrow$1 it assumes for all ‘n’ manifolds as a n- manifold is nothing but a integration of infinite ‘n-1’ – manifolds.

The following Lemma is the fall out of the above result –1

Lemma:

For a manifold of ‘n’-dimensions the corresponding loop shrinks along surface equivalent to‘n-1’-dimensions.

Result 2: An object of ‘n’ dimensions has LSP if & only if corresponding hyper planes of ‘n-1’-dimensions assume LSP.

Now we know that for all 2-manifolds LSP holds. Assuming it to hold for ‘n’ manifolds we can easily show that it holds for ‘n+1’- manifolds as for LSP to hold for ‘n+1’- manifolds implies it holds for ‘n’-manifolds which is assumed to be true. Hence by induction the result is proved for all ‘n’.

In particular we would establish it for a 3-manifold & hence developed a method of visualization of a 3-manifold in particular & n-manifolds in general

Particulars case of a 3-manifold:

As stated before a 2-manifold can be thought to be made of infinite planes. On similar grounds a 3-manifold can be thought to be made of infinite 2-D surfaces.

A 3-manifold many look like this

Following is just a visualization and hence imaginary. We cannot draw a 3-mainfold but for a under standing visualize if as follows from fig.8.

Space for fig.8

The surfaces S1 S2, etc. are elementary 2-D surface and can't be broken down into more elementary surfaces similar to a plane which is the smallest entity for a 2-manifold that has all the properties (topological) of the manifolds and cannot further be reduced.

Mathematically a 2-manifold may be defined as union of infinite 1-manifolds.

$\Longrightarrow M = S1 \cup S2 \ldots .\cup Sn$ *, Here M is a manifold of higher dimension.*

As usual S1,S2 …Sn are manifolds of penultimate dimension of M.

Now from Results 1&2 LSP holds for above manifold if holds for all elements any 2-mainfolds S1 S2 S3..........Sn etc.

$\therefore$ *If LSP holds for all S1 S2 S3..........Sn*

$\Rightarrow$ *All S1 S2 S3.......... etc. are equivalent to a 2- sphere.*

Combination all of there constitute a corresponding 3-sphere.

A 3-sphere may look like fig.8.

Please note that this is just a hypothetical visualization. You may have a different mathematically valid visualization. The main emphasis is on the properties exhibited rather than the physical scaling of the visualization.

Here S1 S2 S3..........Sn are 2-spheres.

$\therefore$ *Poincare's conjecture is true for a 3-manifold in particular from results 1&2 and in general for all n-manifolds through induction.*

The heart of the above proof is that a sphere is an integration of infinite planes having infinite circles one for one plane.

IIIrly a 3-sphere is a integration of infinite 2-D surfaces having infinite sphere in them one for each surface.

Rest is easy extension.

Hence if LSP assumes for all 'n-1' manifold $\Rightarrow$ *1 it assumes for all 'n' manifolds as an n- manifold is nothing but a integration of infinite 'n-1' – manifolds.*

Sole author

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